HW 7 Help

30. ORGANIZE AND PLAN We are given the angular velocity and the time, and we are asked to find the distance that is covered. We can first solve for the angular displacement using Equation 8.3: $\Delta\theta = \omega\Delta t.$ The distance traveled is just the arc length from Equation 8.1: $s = r\Delta\theta.$

Known: $\omega = 4.0 \text{ rev/s}, \quad \Delta t = 1 \text{ min},$ and $r = 0.79 \text{ m}.$

SOLVE First, find the angular displacement:

$$\Delta\theta = \omega\Delta t = (4.0 \text{ rev/s})(1 \text{ min})\left[\frac{60 \text{ s}}{1 \text{ min}}\right] = 240 \text{ rev}$$

Solving for the distance traveled, we'll need to convert to radians:

$$s = r\Delta\theta = (0.79 \text{ m})(240 \text{ rev})\left[\frac{2\pi}{1 \text{ rev}}\right] = 1200 \text{ m}$$

REFLECT This implies the wheel is spinning fast enough to go over a kilometer in one minute, or more precisely 72 kilometers per hour (43 mph). That would be awfully fast for a bicycle, but since the wheel is simply spinning off of the ground, it seems reasonable that it could turn that fast.

38. ORGANIZE AND PLAN We're given the initial angular velocity and the final angular velocity (zero in this case, since she stops spinning), and how long it takes. Therefore, Equation 8.8 can be used to solve for the angular acceleration. For the second part, we want to find $\Delta\theta = \theta - \theta_0$, so either Equation 8.9 or 8.10 should work.

Known: $\omega_0 = 2.50$ rev/s, $\omega = 0$ rev/s, $t = 1.75$ s.

SOLVE (a) First, converting the initial angular velocity to rad/s:

$$\omega_0 = 2.50 \text{ rev/s}\left[\frac{2\pi \text{ rad}}{1 \text{ rev}}\right] = 15.7 \text{ rad/s}$$

Plugging this into Equation 8.8:

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{-15.7 \text{ rad/s}}{1.75 \text{ s}} = -8.98 \text{ rad/s}^2$$

(b) We will solve for $\Delta\theta = \theta - \theta_0$ first with Equation 8.9:

$$\Delta\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = (15.7 \text{ rad/s})(1.75 \text{ s}) + \tfrac{1}{2}(-8.98 \text{ rad/s}^2)(1.75 \text{ s})^2 = 13.7 \text{ rad}$$

As an exercise, let's use Equation 8.10:

$$\Delta\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{-(15.7 \text{ rad/s})^2}{2(-8.98 \text{ rad/s}^2)} = 13.7 \text{ rad}$$

So they agree as they should, but we want the answer in revolutions:

$$\Delta\theta = 13.7 \text{ rad}\left[\frac{1 \text{ rev}}{2\pi \text{ rad}}\right] = 2.18 \text{ rev}$$

REFLECT The acceleration is negative because the skater slows down. If the skater had kept spinning at the same rate for the allotted time, then she would have completed (2.5 rev/s)(1.75 s) = 4.37 rev. Our answer should be less than that (which it is), since the skater slows to a stop. In fact, she completes half as many turns in the stopping case as in the constant spinning case.

40. ORGANIZE AND PLAN We have the initial and final angular velocity and the time, so Equation 8.8 will be needed to find the angular acceleration. For the second part, we can use either Equation 8.9 or 8.10 to find $\Delta\theta = \theta - \theta_0$.

Known: $\omega_0 = 3.40$ rev/s, $\omega = 5.50$ rev/s, $t = 1.30$ s.

SOLVE (a) Let's first convert the angular velocities into rad/s:

$$\omega_0 = 3.40 \text{ rev/s}\left[\frac{2\pi \text{ rad}}{1 \text{ rev}}\right] = 21.4 \text{ rad/s}$$

$$\omega = 5.50 \text{ rev/s}\left[\frac{2\pi \text{ rad}}{1 \text{ rev}}\right] = 34.6 \text{ rad/s}$$

Plugging these values into Equation 8.8:

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{34.6 \text{ rad/s} - 21.4 \text{ rad/s}}{1.30 \text{ s}} = 10.2 \text{ rad/s}^2$$

(b) We will use Equation 8.10 (but 8.9 could be used as well):

$$\Delta\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(34.6 \text{ rad/s})^2 - (21.4 \text{ rad/s})^2}{2(10.2 \text{ rad/s}^2)} = 36.2 \text{ rad}$$

REFLECT The acceleration is positive, since the pulley increases its speed when the gas pedal is pushed.

44. ORGANIZE AND PLAN We are given the period (the time it takes to make a revolution), so we need to use Equation 8.5 to get the angular velocity. Once we have that, we can find the tangential speed using Equation 8.11: $v_t = r\omega$.

Known: $T = 3.20$ s, $d = 1.75$ m.

SOLVE The angular velocity of the wagon wheel is:

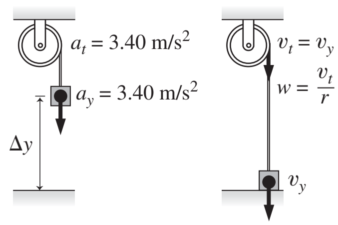
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3.20 \text{ s}} = 1.96 \text{ rad/s}$$

Plugging this into Equation 8.11 gives:

$$v_t = r\omega = (\tfrac{1}{2} \cdot 1.75 \text{ m})(1.96 \text{ rad/s}) = 1.72 \text{ m/s}$$

REFLECT The speed at the rim of the wheel is also the speed that the wagon will move at (assuming the wheel is not slipping). Converting the answer to a more familiar quantity: $v_t = 3.43 \text{ m/s} = 3.87 \text{ mph}$. That certainly seems a reasonable speed for a wagon.

47. ORGANIZE AND PLAN As we see in the figure below, the weight's acceleration induces a tangential acceleration on the pulley through the tension in the string (i.e., $a_y = a_t$). We can convert this into an angular acceleration with Equation 8.12: $\alpha = a_t / r$. For part (b), the falling weight achieves a velocity, v_y, by the time it reaches the ground. This linear velocity is equal to the tangential velocity of the pulley: $v_y = v_t$. There are different ways to go about this, but we will solve first for the weight's velocity using Equation 2.10: $v_y^2 = v_{y0}^2 + 2a_y \Delta y$, and then use this to find the angular velocity from Equation 8.11: $\omega = v_t / r$.



Known: $r = 3.50$ cm, $a_y = 3.40$ m/s^2, $v_{0y} = 0$, $\Delta y = 1.30$ m, where we have defined "down" as the positive
y-direction.

SOLVE (a) The angular acceleration comes directly from the weight's acceleration:

$$\alpha = \frac{a_y}{r} = \frac{3.40 \text{ m/s}^2}{3.50 \text{ cm}}\left[\frac{100 \text{ cm}}{1 \text{ m}}\right] = 97.1 \text{ rad/s}^2$$

(b) The first thing is to solve for the final velocity of the weight:

$$v_y = \sqrt{v_{y0}^2 + 2a_y \Delta y} = \sqrt{2(3.40 \text{ m/s}^2)(1.30 \text{ m})} = 2.97 \text{ m/s}$$

The pulley turns at the same speed, so the angular velocity is:

$$\omega = \frac{v_y}{r} = \frac{2.97 \text{ m/s}}{0.0350 \text{ m}} = 84.9 \text{ rad/s}$$

REFLECT You may wonder why the weight doesn't fall with the gravitational acceleration of 9.8 m/s^2. The string provides a tension that slows the weight's fall. Notice, too, that we chose the down direction to be positive, but we can choose it to be negative, in which case all the answers would have a negative sign.

57. **ORGANIZE AND PLAN** We're only given the saw's initial rotational kinetic energy (Equation 8.15): $K_0 = \frac{1}{2}I\omega_0^2$. The rotation rate drops in half $(\omega = \frac{1}{2}\omega_0)$, while the rotational inertia, *I*, does not change.
Known: $K_0 = 44\text{ J}$, $\omega = \frac{1}{2}\omega_0$.

SOLVE The rotational kinetic energy after the rotation rate drop:

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}I(\frac{1}{2}\omega_0)^2 = \frac{1}{4}(\frac{1}{2}I\omega_0^2) = \frac{1}{4}K_0 = \frac{1}{4}(44\text{ J}) = 11\text{ J}$$

This says that the rotational kinetic energy drops to one-fourth its initial value.

Reflect Because the rotation kinetic energy is proportional to the angular velocity squared, any increase or decrease to the angular velocity will be squared in the rotational kinetic energy.

68. **ORGANIZE AND PLAN** The translational kinetic and rotational energy are the same as in the preceding problem. The only thing that changes is the rotational inertia: is: $I = \frac{2}{5}MR^2$ (from Table 8.4).

SOLVE As before, let's calculate the rotational kinetic energy first:

$$K_{\text{rot}} = \frac{1}{2}(\frac{2}{5}MR^2)(v_{\text{cm}}/R)^2 = \frac{1}{5}Mv_{\text{cm}}^2$$

The fractions of kinetic energy in translational and rotational motion are:

$$\frac{K_{\text{trans}}}{K_{\text{trans}} + K_{\text{rot}}} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{5}} = \frac{5}{7}$$

$$\frac{K_{\text{rot}}}{K_{\text{trans}} + K_{\text{rot}}} = \frac{\frac{1}{5}}{\frac{1}{2} + \frac{1}{5}} = \frac{2}{7}$$

Where the term Mv_{cm}^2 was factored out of the equations.

REFLECT As was already discussed in Example 8.11, the solid sphere has slightly less rotational inertia than the solid cylinder, which implies that a smaller fraction of its energy goes into rotation. This leaves more for translation, so a sphere will beat a cylinder in rolling down an incline.

74. ORGANIZE AND PLAN The torque is given by Equation 8.16: $\tau = rF\sin\theta$. We're told the radius, the force applied and the angle. In the first case, the angle is 90º (tangential means in the same direction as the motion), whereas in the second case it's 45º.

Known: $r = 1.05\text{ m}$, $F = 23.0\text{ N}$, $\theta = 90^\circ, 45^\circ$.

SOLVE Plugging the values in for the two cases:

(a)

$$\tau = (1.05\text{ m})(23.0\text{ N})\sin 90^\circ = 24.2\text{ N}\cdot\text{m}$$

(b)

$$\tau = (1.05\text{ m})(23.0\text{ N})\sin 45^\circ = 17.1\text{ N}\cdot\text{m}$$

REFLECT It's clear that pushing in the tangential direction is the most effective way to generate torque. Any force exerted in the direction perpendicular to the motion will be wasted.

82. ORGANIZE AND PLAN We need to redo what we did in the preceding problem, but this time include the meter stick's weight, which is a downward force at the midpoint (50-cm mark).

Known: $x_1 = 35\text{ cm},\ m_1 = 0.20\text{ kg},\ x_2 = 75\text{ cm},\ m_2 = 0.40\text{ kg},\ x_s = 50\text{ cm},\ M_s = 0.15\text{ kg}.$

SOLVE Defining the positive directions as before, the net force and net torque are:

$$F_{\text{net}} = n_f - m_1 g - m_2 g - M_s g = 0$$
$$\tau_{\text{net}} = r_2 m_2 g + r_s M_s g - r_f n_f = 0$$

We have chosen the axis to be the 35-cm mark, so the torque of the first mass is zero, and the radii are: $r_2 = 40\text{ cm},\ r_s = 15\text{ cm},$ and $r_f = x_f - 35\text{ cm}.$ Solving for n_f in the first equation and substituting it into the second equation gives:

$$r_f = \frac{r_2 m_2 g + r_s M_s g}{m_1 g + m_2 g + M_s g} = \frac{(40\text{ cm})(0.40\text{ kg}) + (15\text{ cm})(0.15\text{ kg})}{(0.20\text{ kg}) + (0.40\text{ kg}) + (0.15\text{ kg})} = 24\text{ cm}$$

This means the fulcrum needs to be put at: $x_f = r_f + 35\text{ cm} = 59\text{ cm}.$

REFLECT By including the mass of the meter stick in this problem, we increase the amount of mass to the left of the fulcrum (as it was placed in the previous problem). It makes sense that the fulcrum will have to be moved to the left to compensate (i.e., from the 62-cm mark to the 59-cm mark).

94. **ORGANIZE AND PLAN** Imagine that the electron is circling its proton in a counter-clockwise direction as seen from above. The velocity we're given is the tangential velocity, so we'll need to use Equation 8.11: $\omega = v_t / r$ to find the angular velocity. The electron can be treated as a point particle moving in a circle, so the rotational inertia is: $I = mr^2$. The mass of the electron can be found in the Appendix.

Known: $v_t = 2.18 \times 10^6$ m/s, $r = 5.29 \times 10^{-11}$ m, $m = 9.11 \times 10^{-31}$ kg.

SOLVE The magnitude of the angular momentum is:

$$\begin{aligned} L &= I\omega = (mr^2)(v_t / r) = mv_t r \\ &= (9.11 \times 10^{-31} \text{ kg})(2.18 \times 10^6 \text{ m/s})(5.29 \times 10^{-11} \text{ m}) = 1.05 \times 10^{-34} \text{ J} \cdot \text{s} \end{aligned}$$

The direction will be up if the electron is going in a counter-clockwise direction.

REFLECT The angular momentum of the electron is an important quantity in quantum mechanics. It comes as specific multiples of the so-called reduced Planck constant: $\hbar = 1.05 \times 10^{-34}$ J·s. The electron in this example has the lowest angular momentum: $L = 1 \cdot \hbar$.